

Linear Stochastic Estimation and Pressure Field Calculation from Velocity Field Data

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In aeroacoustic studies of bluff bodies, such as circular cylinders, the structure and evolution of the wake flow play a crucial role in understanding both the aerodynamic and aeroacoustic behavior, especially those associated with the Kármán vortex shedding. However, the common velocity field measurement technique, Particle Image Velocimetry (PIV), is usually non-time-resolved (NTR) and therefore unable to reveal the temporal dynamics of the wake. To overcome this limitation, researchers have developed the linear stochastic estimation (LSE) method [1] to reconstruct estimated time-resolved (TR) velocity fields by making use of the correlation between PIV snapshots and certain reference signals that are time-resolved.

The availability of TR velocity fields enables the calculation of pressure fields from velocity field data, which provides unsteady surface pressure data for far-field noise prediction using Curle's analogy [2]. The pressure field is obtained by solving a Poisson problem [3, 2, 4] established from the unsteady Reynolds-Averaged Navier-Stokes (URANS) equations, specifically the continuity and the momentum equations.

This study explores the application of the LSE and pressure-field calculation methods to velocity fields of a two-dimensional circular cylinder measured by NTR PIV. The study also identifies key implementation factors that affect the accuracy of these methods. For the LSE approach, the velocity fields are first decomposed through proper orthogonal decomposition (POD) and represented by their temporal coefficients a_i so that the task is converted to estimating the TR temporal POD coefficients \hat{a}_i .

The TR reference signal used is the unsteady surface pressure measured using the remote microphone probe (RMP) technique at discrete pressure ports p_k . The time-delayed cross-correlation between the NTR temporal POD coefficients and the TR unsteady pressure $R_{a_i p_k}$ is first calculated, before taking Fourier transform to obtain the cross-spectral density (CSD) $S_{a_i p_k}$. The CSD between two TR pressure signals $S_{p_j p_k}$ is calculated in the same manner. A system of linear equations $S_{p_j p_k} A_{ij} = S_{a_i p_k}$ can then be formed to solve for the transfer function A_{ij} from which the TR temporal POD coefficients (in frequency domain) can be calculated as $\tilde{a}_i = A_{ij} p_j$. Taking the inverse Fourier transform of \tilde{a}_i results in the estimated time-domain coefficients \hat{a}_i with which low-order TR velocity fields can be reconstructed.

For pressure field calculation, URANS equations are used to establish a Poisson equation $\frac{\partial^2 \bar{p}}{\partial x_i^2} = S$, where $S = \frac{\partial}{\partial x_i} \left(- \left(\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} \right) - \frac{\partial \overline{u'_i u'_j}}{\partial x_j} + \frac{\partial}{\partial x_j} \left(\nu \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \right) \right)$. The symbol $(\bar{\cdot})$ denotes ensemble averaging. Ensemble-averaged quantities are used because the TR velocity fields reconstructed via LSE are essentially low-pass filtered, coherent part of the flow. Dirichlet boundary conditions are applied on boundaries outside the wake region, where the unsteady Bernoulli's equation can be employed to calculate the pressure values. Neumann boundary conditions are applied on the remaining boundaries. To solve the Poisson equation, the open-source package OpenFoam [5] is implemented. The pressure calculation methodology is verified by using computational fluid dynamics (CFD) data and by comparing the calculated pressure with the experimental measurements obtained using the RMP technique.

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