# On the modified $(n+1)$D Laplacian for smooth pressure reconstruction based on time-resolved Velocimetry

## Why LaVision's (3+1)D pressure solvers is fundamentally and unnecessarily problematic?

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Smooth reconstruction of pressure fields from time-resolved image velocimetry measurements typically follows two approaches: (1) reducing noise in the measured velocity field and/or (2) applying a smoothing technique to the pressure reconstruction. The pressure solver introduced in DaVis 10.0 (LaVision) follows the latter approach by incorporating a "modified Poisson equation": $\nabla^2 p + \xi^2 \frac{\partial^2 p}{\partial t^2} = f(\boldsymbol{u}(t))$, where $p$ is the pressure field, $\boldsymbol{u}(t)$ is the velocity field measured by time-resolved image velocimetry, and $\xi^2$ is a tunable parameter to control the solver's diffusive and smoothing behavior in time.

Sakib et al. (2021) used the four-dimensional solver (implemented in DaVis 10.2) to reconstruct the pressure field from the velocity field of an impinging synthetic jet measured by a time-resolved tomographic PIV system. The study showed that while the 4D solver removes the high-frequency noise, the reconstructed pressure fields sometimes suffer from drift in time, which is problem-dependent (e.g., different non-dimension frequency of the synthetic jet leads to different levels of temporal drift in the reconstruction). Later, in Sakib et al. (2023), a parametric study was performed based on the same impinging synthetic jet data. The study demonstrated that parameter tuning is important for both the smooth reconstruction of the pressure field and the temporal drift, but it is difficult to select an optimal $\xi^2$.

In our study, we first provide analytical intuition for the smoothing effect of the modified pressure Poisson equation using functional analysis and Fourier analysis. Then we quantify the error propagation dynamics of the modified Poisson solver by introducing an upper bound on the error in the reconstructed pressure fields. Finally, we discuss the potential drift due to the tricky boundary condition setup and the partitioning in time, which is an optional strategy used in LaVision's current 4D pressure solver to reduce computational costs. Our analysis and validation show that when an $(n+1)$D solver is used, careful choice of $\xi^2$ and partition in time is needed for smooth and accurate pressure field reconstruction.

Although parameter tuning can partially rectify the problem of introducing errors in the reconstructed pressure fields, the modified Poisson solver is unnecessarily flawed in several ways: i) violating the pressure Poisson equation derived from the Navier-Stokees equations, ii) causing higher computational cost with one extra dimension, and iii) accumulation of error causing pressure drift over time (Sakib et al., 2023; Zhang et al., 2024). In Sakib et al. (2025), the experimental part of our series study, we demonstrated that the smoothing behavior results from the second-order low-pass filter effect of the inversion of the modified Laplacian. Using a second-order low-pass filter to suppress the noise of the constructed pressure field from an $n$D solver is perhaps a more efficient and appropriate method for a smooth pressure field reconstruction without solving a 4D partial differential equation.

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## References

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